

2.13

Sequences

A **sequence is a function** whose domain is the set of positive integers.

A sequence never ends. The numbers in the list are called the **terms** of the sequence.

Recursive Formulas: A way of defining a sequence is to assign a value to the first (or the first few) term(s) and specify the n th term by a formula or equation that involves one or more of the terms preceding it. Sequences defined this way are said to be defined **recursively** and the rule or formula is called a **recursive formula**.

Example: $s_1 = 1 \quad s_n = 4s_{n-1}$

$$S_2 = 4 \cdot S_{2-1}$$

$$S_2 = 4 \cdot S_1$$

$$S_2 = 4 \cdot 1 = 4$$

So, the sequence would be 1, 4, 16, 64, ...

The Fibonacci sequence: 1, 1, 2, 3, 5,... can be defined recursively by:

$$S_1 = 1, \quad S_2 = 1 \quad \text{for } n > 2$$

$$S_n = S_{n-2} + S_{n-1}$$

Arithmetic Sequences

When the difference between successive terms of a sequence is always the same number, the sequence is called **arithmetic**. An **Arithmetic sequence** may be defined **recursively** as:

$$a_1 = a, \quad a_n - a_{n-1} = d \quad \text{or as}$$

$$a_1 = a, \quad a_n = a_{n-1} + d \quad \text{where } a = a_1 \text{ and } d \text{ are real numbers. (Recursive Formula)}$$

The number **a** is the **first term** and the number **d** is called the **common difference**.

For an arithmetic sequence $\{a_n\}$ whose first term is **a** and whose common difference is **d** , the n th term is determined by the **explicit formula**...

$$a_n = a + (n - 1)d$$

Example:

For the following arithmetic sequence, find a) the common difference, b) the tenth term, c) a recursive rule for the n th term, and d) an explicit rule for the n th term.

1. -6, -2, 2, 6, 10, ...

a) The difference between successive terms is 4.

b) $a_{10} = -6 + (10 - 1)(4) = 30$ (using explicit formula above)

c) The sequence is defined recursively by $a_1 = -6$ and $a_n = a_{n-1} + 4$ for all $n \geq 2$.

d) The sequence is defined explicitly by $a_n = -6 + (n - 1)(4) = 4n - 10$.

Geometric Sequences

When the ratio of successive terms of a sequence is always the same nonzero number, the sequence is called **geometric**. A **geometric sequence** may be defined **recursively** as

$$a_1 = a, \quad \frac{a_n}{a_{n-1}} = r \quad \text{or as} \quad a_1 = a, \quad a_n = r \bullet a_{n-1}$$

Where $a_1 = a$ and $r \neq 0$ are real numbers. The number **a** is the **first term**, and the nonzero number **r** is called the **common ratio**.

The terms of a geometric sequence with first term a and common ratio r follow the pattern:

$a, ar, ar^2, ar^3 \dots$

For a geometric sequence $\{a_n\}$ whose first term is **a** and whose common ratio is **r** , the n th term is determined by the **explicit formula**:

$$a_n = ar^{n-1}, \quad r \neq 0$$

Example:

For the following geometric sequence find a) the common ratio, b) the tenth term, c) a recursive rule for the n th term, and d) an explicit rule for the n th term.

2. 3, 6, 12, 24, 48, ...

a) The ratio between successive terms is 2.

b) $a_{10} = 3 \cdot 2^{10-1} = 3 \cdot 2^9 = 1536$

c) The sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1}$ for $n \geq 2$.

d) The sequence is defined explicitly by $a_n = 3 \cdot 2^{n-1}$.